1. Klausur ALP I
Lösungen zur Aufgabe 3

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8. April 2003

Für alle Versionen muß in Teil a) folgendes Schema eingehalten werden:

$$\begin{aligned} f(0) &= g() \\ f(n+1) &= h(n, f(n)) \end{aligned}$$

Ich gebe dann jeweils nur noch g und h an. Außerdem werden die Funktionen `add`, `mul` und `msub` als gegeben betrachtet.

Version a

a)

Mathematisch:

$$\begin{aligned} f(0) &= 0 \\ f(n+1) &= (n+1)^2 + f(n) \end{aligned}$$

Primitiv Rekursiv:

$$\begin{aligned} g() &= k_0^0() \\ h(a,b) &= \mathtt{add}(\mathtt{mul}(s(a), s(a)), b) \end{aligned}$$

b)

$$\begin{aligned} w(x) &= \min\{y | y \in \mathbb{N} \wedge w(y,x) = 0\} \\ w(y,x) &= x - y^3 \\ w(y,x) &= \mathtt{msub}(\mathtt{P}_2^2(y,x), \mathtt{hoch3}(\mathtt{P}_1^2(y,x))) \\ \mathtt{hoch3}(x) &= \mathtt{mul}(\mathtt{mul}(\mathtt{P}_1^1(x), \mathtt{P}_1^1(x)), \mathtt{P}_1^1(x)) \end{aligned}$$

Version b

a)

Mathematisch:

$$\begin{aligned} f(0) &= 1 \\ f(n+1) &= (n+1) + 1 + f(n) \end{aligned}$$

Primitiv Rekursiv:

$$\begin{aligned} g() &= k_1^0() \\ h(a,b) &= \mathtt{add}(s(s(a)), b) \end{aligned}$$

b)

$$\begin{aligned} w(x) &= \min\{y | y \in \mathbb{N} \wedge w(y,x) = 0\} \\ w(y,x) &= x^2 - (y^2 - 1) \\ w(y,x) &= \mathtt{msub}(\mathtt{mul}(\mathtt{P}_2^2(y,x), \mathtt{P}_2^2(y,x)), \mathtt{yQuadratMin1}(\mathtt{P}_1^2(y,x))) \\ \mathtt{yQuadratMin1}(y) &= \mathtt{msub}(\mathtt{mul}(\mathtt{P}_1^1(y), \mathtt{P}_1^1(y)), k_1^1(y)) \end{aligned}$$

Version c

a)

Mathematisch:

$$\begin{aligned} f(0) &= 0 \\ f(n+1) &= 3(n+1) + f(n) \end{aligned}$$

Primitiv Rekursiv:

$$\begin{aligned} g() &= k_0^0() \\ h(a,b) &= \mathtt{add}(\mathtt{mul}(k_3^1(a), s(a)), b) \end{aligned}$$

b)

$$\begin{aligned} w(x) &= \min\{y \mid y \in \mathbb{N} \wedge w(y,x) = 0\} \\ w(y,x) &= 4x - y^2 \\ w(y,x) &= \mathtt{msub}(\mathtt{mul}(k_4^2(y,x), \mathtt{P}_2^2(y,x)), \mathtt{mul}(\mathtt{P}_1^2(y,x), \mathtt{P}_1^2(y,x))) \end{aligned}$$

Version d

a)

Mathematisch:

$$\begin{aligned} f(0) &= 0 \\ f(n+1) &= (n+1) + f(n) \end{aligned}$$

Primitiv Rekursiv:

$$\begin{aligned} g() &= k_0^0() \\ h(a,b) &= \mathtt{add}(s(a), b) \end{aligned}$$

b)

$$\begin{aligned} w(x) &= \min\{y \mid y \in \mathbb{N} \wedge w(y,x) = 0\} \\ w(y,x) &= x - y^4 \\ w(y,x) &= \mathtt{msub}(\mathtt{P}_2^2(y,x), \mathtt{hoch4}(\mathtt{P}_1^2(y,x))) \\ \mathtt{hoch4}(x) &= \mathtt{mul}(\mathtt{mul}(\mathtt{P}_1^1(x), \mathtt{P}_1^1(x)), mul(\mathtt{P}_1^1(x), \mathtt{P}_1^1(x))) \end{aligned}$$